Int J Theor Phys (2008) 47: 2697–2705
DOI 10.1007/s10773-008-9708-z

Quantum Effect of the Exciton Intensity in a Semiconductor Microcavity

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Received: 10 January 2008 / Accepted: 12 March 2008 / Published online: 21 March 2008

Abstract We have studied the collapses and revivals of the exciton intensity in a semiconductor microcavity under the resonant case. It is found that, when the excitons are initially in the number state, the exciton intensity exhibits the periodic oscillation if the dissipation parameters equal zero, but if there exists the dissipation, the damped oscillation appears. Whether there exists the dissipation or does not, the width of oscillation decreases with the increment of the atom numbers. When the excitons are initially in the coherent state, the width of the oscillation decreases with the increment of the dissipation.

Keywords Collapse and revival effect · Exciton emission

1 Introduction

With the development of crystal growth techniques, people can now fabricate multi-dimensional confined nano-structure materials, such as quantum wells, quantum lines and quantum dots. Some interesting phenomena not observed in bulk material may take place within these systems. The optical properties of a microcavity containing semiconductor quantum wells have been studied intensely in the past years [1] since the first observation of polaritons splitting in the stronger-coupling regime [2, 3]. Recently, Jin et al. [4] have studied the collapses and revivals of light intensity in a semiconductor microcavity. In this

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paper, we shall study the collapses and revivals of the exciton intensity in a semiconductor microcavity.

2 Model

The model we considered here is a microcavity containing a semiconductor quantum well embedded in a high finesse cavity. We assume that the cavity and the quantum well are ideal, and they are in an extremely low temperature situation. The quantum well interacts with cavity field via exciton, which is an electron-hole pair bound by the Coulomb interaction. The exciton and the photon modes are quantized along the direction normal to the microcavity, so we will consider the lowest-order mode in this direction. The excitons with in-plane wave vector $\mathbf{K}$ may only be dressed by the photons with the same wave vector due to the transitional invariance in the plane of the microcavity.

To further simplify the model, we will consider only one mode of photons with wave vector $\mathbf{K} = \mathbf{0}$ and frequency ω_c very close to the lowest $n = 1s$ exciton energy level [5, 6]. In fact, at extremely low temperature, the thermal momentum of the excitons is so small that the thermalized excitons can be neglected [7, 8]. Combining the above considerations and neglecting the spin degrees of freedom, one can write an effective Hamiltonian for the coupled exciton-photon system as [5, 6]

$$\hat{H} = \omega_c \hat{a}^\dagger \hat{a} + \omega_{ex} \hat{b}^\dagger \hat{b} + g(\hat{a}^\dagger \hat{b} + \hat{b}^\dagger \hat{a}) + A \hat{b}^\dagger \hat{b}^\dagger \hat{b}\hat{b} - B(\hat{b}^\dagger \hat{b}^\dagger \hat{b}\hat{a} + \hat{a}^\dagger \hat{b}^\dagger \hat{b}\hat{b}), \tag{1}$$

where $\hat{b}^\dagger(\hat{b})$ are creation (annihilation) operators of the excitons with frequency ω_{ex}, and $\hat{a}^\dagger(\hat{a})$ are creation (annihilation) operators of the cavity field with frequency ω_c. Here we assume that both of them obey the bosonic commutation relation $[\hat{b}, \hat{b}^\dagger] = [\hat{a}, \hat{a}^\dagger] = 1$.

If we introduce polariton operators $p_1(t) = -va(t) + ub(t)$ and $p_2(t) = ua(t) + vb(t)$ satisfying commutation relations $[p_i, p_j^\dagger] = \delta_{ij}$ $(i, j = 1, 2)$, then Hamiltonian (1) can be rewritten as [4]

$$H_{eff} = \sum_{j=1,2} \omega_j p_j^\dagger p_j + A_{11} p_1^\dagger p_1^\dagger p_1 p_1 + A_{22} p_2^\dagger p_2^\dagger p_2 p_2 + 2A_{12} p_1^\dagger p_2^\dagger p_2 p_1, \tag{2}$$

where

$$u = \sin\theta, \qquad v = \cos\theta, \qquad \tan 2\theta = -2g/\delta, \qquad \delta = \omega_c - \omega_{ex}, \tag{3}$$

$$\omega_j = \frac{1}{2}[\omega_c + \omega_{ex} + (-1)^j \Delta] \quad (j = 1, 2), \qquad \Delta = \omega_2 - \omega_1 = \sqrt{\delta^2 + 4g^2}, \tag{4}$$

$$\begin{aligned} &A_{11} = u^3(Au + 2Bv), \qquad A_{22} = v^3(Av - 2Bu), \\ &A_{12} = A_{21} = 2uv[Auv - B(u^2 - v^2)]. \end{aligned} \tag{5}$$

In (2), we have neglected some terms proportional to $p_1^\dagger p_1^\dagger p_2 p_2$, $p_1^\dagger p_1^\dagger p_1 p_2$ and their Hermitian conjugate terms, which describe scattering processes between the two polariton branches and destroy particle-number conservation within each polariton branch.

The formal solutions of the Heisenberg equation for the polariton operators $p_j(t)$ are [4]

$$p_j(t) = \exp\left\{-i\left[\omega_j - i\gamma_j/2 + 2\sum_{k=1}^{2} A_{jk} p_k^\dagger p_k\right] t\right\} p_j, \tag{6}$$

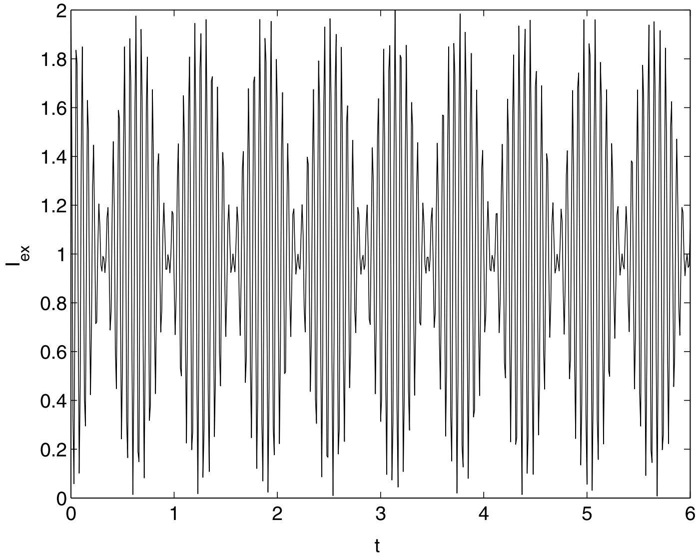

Fig. 1 Diagram of the exciton intensity for the number state as a function of time t in the resonant case. $g = 1000\gamma$, $A = 0.01g$, $B = 0$. $N = 2$, $\gamma_1 = \gamma_2 = 0$

where $\gamma_1(\gamma_2)$ is the natural linewidth of the lower- (upper-) branch of the polariton. The initial time operators (say, $p_j(0)$) have been written in the compact form (p_j). In the below, unless we specify otherwise, all the compact form operators stand for the operators at time $t = 0$.

3 Quantum Effect of Exciton Intensity in a Semiconductor Microcavity

In the following, we will study the quantum effect of exciton intensity in a semiconductor microcavity. We first introduce the Schwinger's angular momentums $J_z = \frac{1}{2}(b^\dagger b - a^\dagger a)$ and the ladder operator $J_+ = (J_-)^\dagger = b^\dagger a$, which satisfying the commutation relations $[J_+, J_-] = 2J_z$, and $[J_z, J_\pm] = \pm J_\pm$. So the quantum states of the coupled exciton-photon system can be written in terms of the angular momentum states $|j,m\rangle = (b^\dagger)^{j-m}(a^\dagger)^{j+m}/\sqrt{(j-m)!(j+m)!}|0\rangle$, which is a direct product of two number states with $(j-m)$ excitons in the quantum well and $(j+m)$ photons in the cavity, respectively.

3.1 Number State Case

If the excitons are initially in vacuum state and the photons are initially in a number state $|\phi(0)\rangle_c = |N\rangle_c$, then the initial state of the total system may be written in terms of the angular momentum states as $|\psi(0) = |j,j\rangle$ with $j = N/2$. The exciton intensity can be expressed as

$$I_{ex}(t) = \langle\psi(0)|b^\dagger(t)b(t)|\psi(0)\rangle = u^2\langle p_1^\dagger p_1\rangle + v^2\langle p_2^\dagger p_2\rangle + uv[\langle p_1^\dagger(t)p_2(t)\rangle + c.c]$$

$$= \frac{N}{2}(1+\cos^2 2\theta) + \frac{\alpha}{2}\sin 2\theta\Bigg[-\sum_{m=-j}^{j}\sqrt{(j+m+1)(j-m)}e^{i2mt(2A_{12}-A_{11}-A_{22})}$$

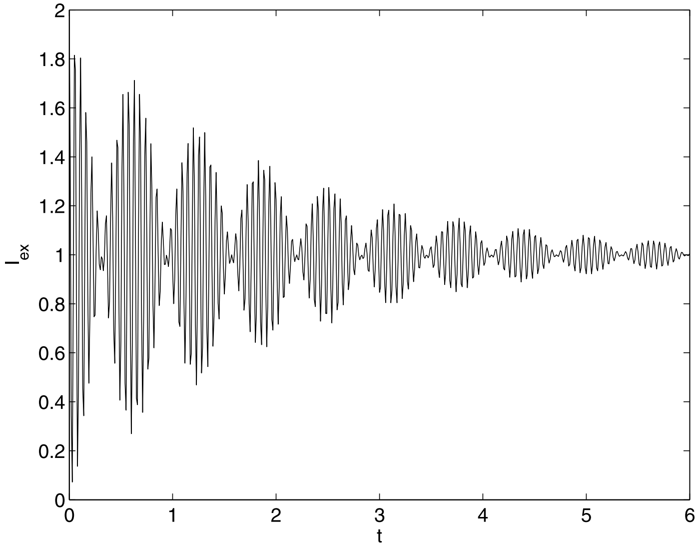

Fig. 2 Diagram of the exciton intensity for the number state as a function of time t in the resonant case. $g = 1000\gamma$, $A = 0.01g$, $B = 0$. $N = 2$, $\gamma_1 + \gamma_2 = \gamma = 1$

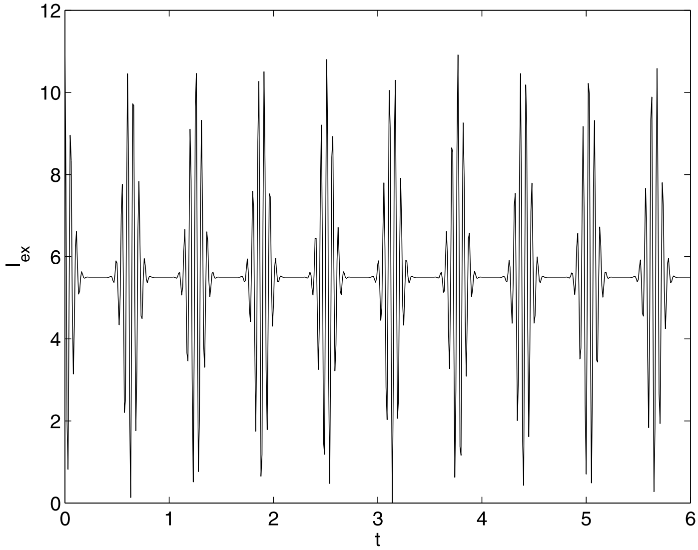

Fig. 3 Diagram of the exciton intensity for the number state as a function of time t in the resonant case. $g = 1000\gamma$, $A = 0.01g$, $B = 0$. $N = 11$, $\gamma_1 = \gamma_2 = 0$

$$\times \langle jj|e^{-2i\theta J_y}|jm\rangle\langle jm+1|e^{2i\theta J_y}|jj\rangle + c.c \Bigg], \tag{7}$$

where $\alpha = e^{i[(A_{11}-A_{22})\langle N\rangle+\omega_1-\omega_2]t}e^{-(\gamma_1+\gamma_2)t/2}$ with N is the initial excitons number in the microcavity, and θ is defined by the relation $\tan 2\theta = -2g/\delta$.

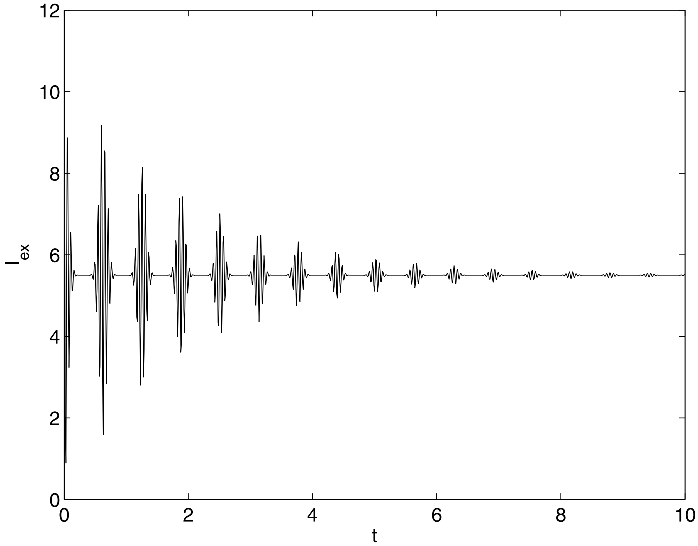

Fig. 4 Diagram of the exciton intensity for the number state as a function of time t in the resonant case. $g = 1000\gamma$, $A = 0.01g$, $B = 0$. $N = 11$, $\gamma_1 + \gamma_2 = \gamma = 1$

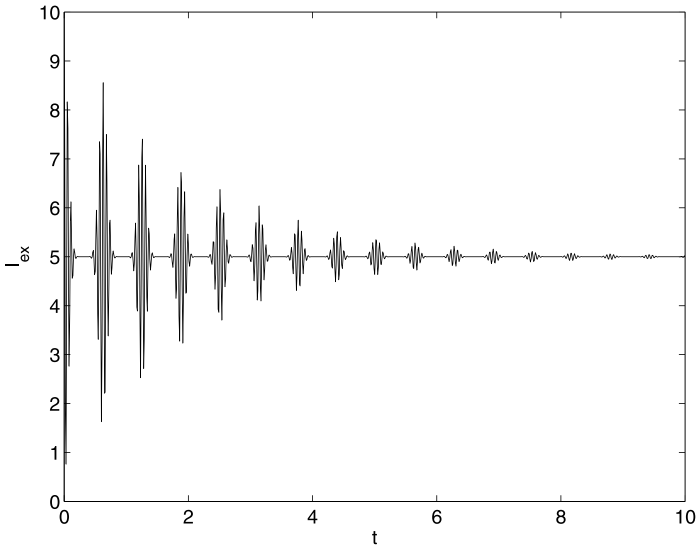

Fig. 5 Diagram of the exciton intensity for the number state as a function of time t in the resonant case. $g = 1000\gamma$, $A = 0.01g$, $B = 0$. $N = 10$, $\gamma_1 + \gamma_2 = \gamma = 1$

Figures 1–6 show the exciton intensity for the number state as a function of time t in the resonant case ($\delta = 0$, i.e., $\theta = \pi/4$). Figures 5–6 show the exciton intensity on $B = 0$ and $B = 300A$. It is found that, when the dissipation parameters equal zero, the exciton intensity

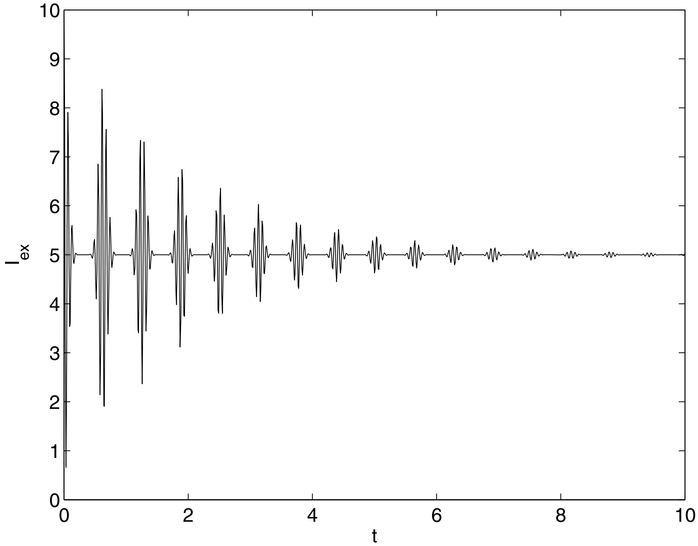

Fig. 6 Diagram of the exciton intensity for the number state as a function of time t in the resonant case. $g = 1000\gamma$, $A = 0.01g$, $B = 300A$. $N = 10$, $\gamma_1 + \gamma_2 = \gamma = 1$

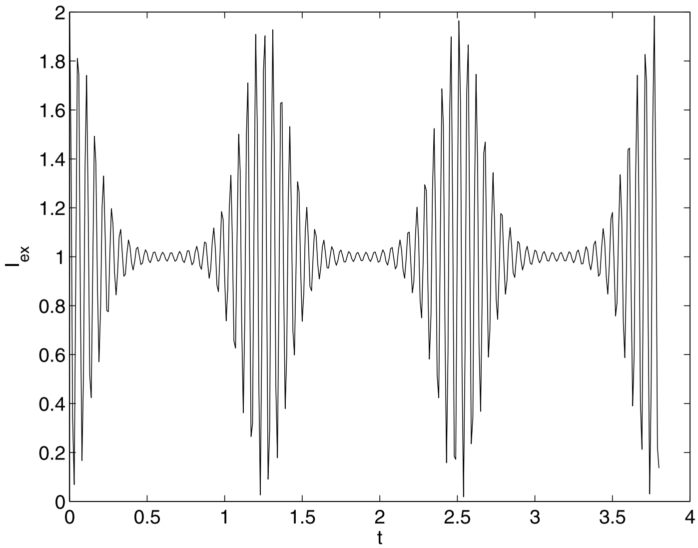

Fig. 7 Diagram of the exciton intensity for the coherent state as a function of time t in the resonant case. $g = 1000\gamma$, $A = 0.01g$, $B = 0$: $\langle N \rangle = 2$, $\gamma_1 = \gamma_2 = 0$

exhibits the periodic oscillation. But if does not, the damped oscillation appears, the width of oscillation decreases with the increment of the atom numbers, and the maximums of the exciton intensity increase rapidly with the increment of the atom numbers.

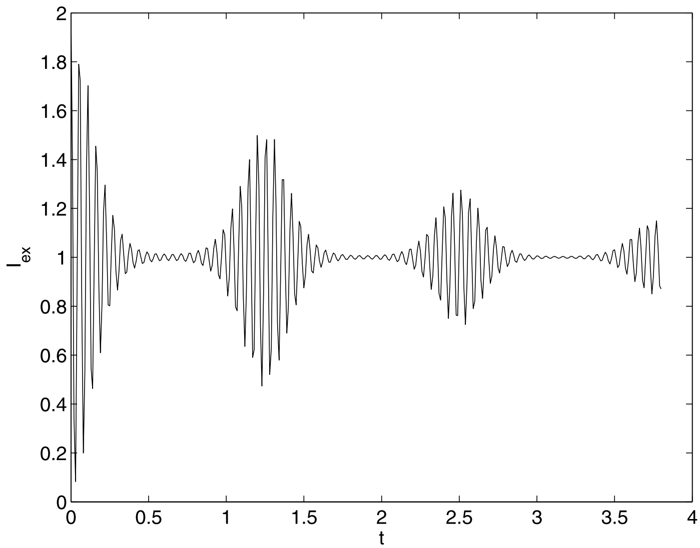

Fig. 8 Diagram of the exciton intensity for the coherent state as a function of time t in the resonant case. $g = 1000\gamma$, $A = 0.01g$, $B = 0$: $\langle N \rangle = 2$, $\gamma_1 + \gamma_2 = \gamma = 1$

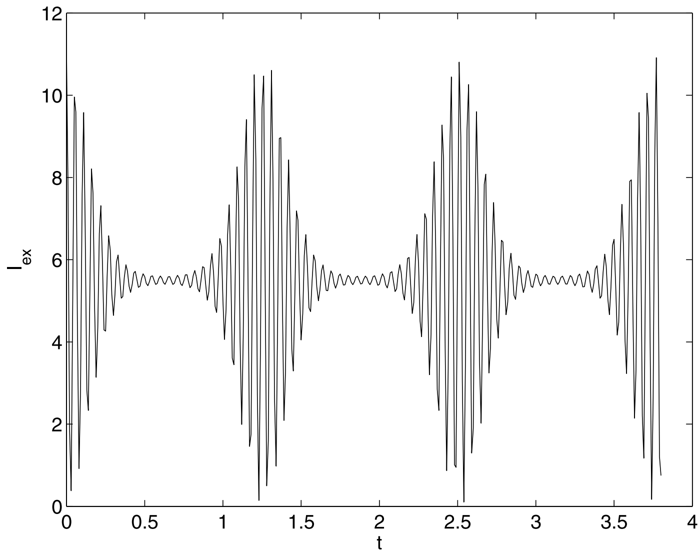

Fig. 9 Diagram of the exciton intensity for the coherent state as a function of time t in the resonant case. $g = 1000\gamma$, $A = 0.01g$, $B = 0$: $\langle N \rangle = 11$, $\gamma_1 = \gamma_2 = 0$

3.2 Coherent State Case

We now consider the case that the excitons are initially in vacuum state and the photons are initially in coherent state. The coherent state is characterized by $\beta = \sqrt{\langle N \rangle} e^{i\phi}$ with the average photon number $\langle N \rangle = |\beta|^2$ and the initial phase ϕ. Then the initial state of the total

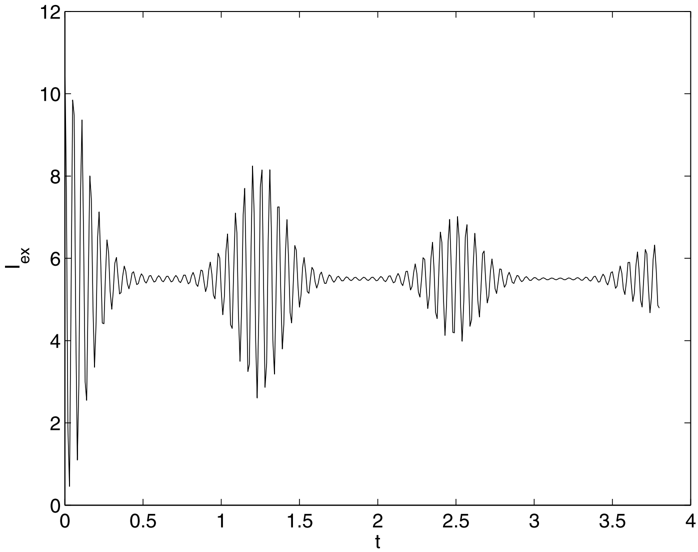

Fig. 10 Diagram of the exciton intensity for the coherent state as a function of time t in the resonant case. $g = 1000\gamma$, $A = 0.01g$, $B = 0$: $\langle N\rangle = 11$, $\gamma_1 + \gamma_2 = \gamma = 1$

system $|\psi(0)\rangle = |0\rangle_{ex}|\beta\rangle_c$ can be written as

$$|\psi(0)\rangle = e^{-|\beta|^2/2}\sum_{j=0}^{\infty}\frac{\beta^{2j}}{\sqrt{(2j)!}}|jj\rangle. \tag{8}$$

The exciton intensity can be expressed as

$$\begin{aligned} I_{ex}(t) &= \langle\psi(0)|b^\dagger(t)b(t)|\psi(0)\rangle = u^2\langle p_1^\dagger p_1\rangle + v^2\langle p_2^\dagger p_2\rangle + uv[\langle p_1^\dagger(t)p_2(t)\rangle + c.c] \\ &= \frac{|\beta|^2}{2}\sin^2 2\theta + \frac{\alpha}{2}\sin 2\theta e^{-|\beta|^2}\Bigg[-\sum_{j=0}^{\infty}\sum_{m=-j}^{j}\sqrt{(j+m+1)(j-m)}\frac{(|\beta|^2)^{2j}}{(2j)!} \\ &\quad\times e^{i2mt(2A_{12}-A_{11}-A_{22})}\langle jj|e^{-2i\theta J_y}|jm\rangle\langle jm+1|e^{2i\theta J_y}|jj\rangle + c.c\Bigg]. \end{aligned} \tag{9}$$

Figures 7–10 show the exciton intensity for the coherent state as a function of time t in the resonant case ($\delta = 0$, i.e., $\theta = \pi/4$). We find that, when the dissipation parameters equal zero, the exciton intensity exhibits the periodic oscillation. But if does not, the damped oscillation appears, the maximums of the exciton intensity increase rapidly with the increment of the atom numbers, and the width of the oscillation decreases with the increment of the dissipation.

4 Conclusions

In conclusion, we have studied the collapses and revivals of the exciton intensity in a semiconductor microcavity under the resonant case. We find that when the excitons are initially in the number state, the exciton intensity exhibits the periodic oscillation if the dissipation parameters equal zero, but if there exists the dissipation, the damped oscillation appears. Whether there exists the dissipation or does not, the width of oscillation decreases with the increment of the atom numbers. When the excitons are initially in the coherent state, the width of the oscillation decreases with the increment of the dissipation.

Acknowledgements This work was supported by the Beijing NSF under Grant No. 1072010.

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